12th EADI General Conference

Global Governance for Sustainable Development

The Need for Policy Coherence and New Partnerships

Counting and Multidimensional Poverty Measurement

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Abstract

This paper proposes a new methodology for multidimensional poverty measurement consisting of: (i) an identification method ρk that extends the traditional intersection and union approaches, and (ii) a class of poverty measures $M\alpha$ that satisfies a range of desirable properties including decomposability. Our identification step makes use of two forms of cutoffs: first, a cutoff within each dimension to determine whether a person is deprived in that dimension; second, a cutoff across dimensions that identifies the poor by counting the number of dimensions in which a person is deprived. The aggregation step employs the FGT measures, appropriately adjusted to account for multidimensionality. The identification method is particularly well suited for use with ordinal data, as is the first of our measures, the adjusted headcount ratio. We provide illustrative examples using data from Indonesia and the US to show how our methodology might be used in practice.

JEL Classifcation: I3, I32, D63, O1

Keywords: poverty measurement, multidimensional poverty, capability approach, deprivation, identification, poverty indices, FGT measures, decomposability, ordinal, cardinal, relative weights, axiomatic structure, freedom.

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1. INTRODUCTION

MULTIDIMENSIONAL POVERTY has captured the attention of researchers and policymakers alike due, in part, to the compelling conceptual writings of Amartya Sen[*] and the unprecedented availability of relevant data. A key direction for research has been the development of a coherent framework for measuring poverty in the multidimensional environment that is analogous to the set of techniques developed in unidimensional space.

Much attention has been paid to the *aggregation* step in poverty measurement through which the data are combined into an overall indicator of multidimensional poverty. The major contributions have developed an array of multidimensional poverty measures and clarified the axioms they satisfy, primarily by extending well-established unidimensional poverty measures and axioms in new and interesting ways.[†] However each of the aggregation techniques relies on a prior *identification* step – namely, 'who is poor?' Considerably less attention has been given to this important component of a poverty methodology.

Identification is implicit in all poverty measures, although it is mainly discussed in measures that first aggregate across dimensions of deprivation at the individual level, then aggregate across individuals. At present there are three main approaches to identifying the poor in a multidimensional setting. One is the 'unidimensional' approach, through which the multiple indicators of wellbeing are combined into a single aggregate variable, and a person is identified as poor when the variable falls below a certain cutoff level. This method of identification takes into account dimensional deprivations – but only inasmuch as they affect the aggregate indicator. There is minimal scope for valuing dimensional deprivations *per se*, which is often viewed as an essential characteristic of a multidimensional approach. A second is the 'union' approach, which regards someone who is deprived in a single dimension as poor in the multidimensional sense. This is generally acknowledged to be overly inclusive and may lead to exaggerated estimates of poverty. The third main approach is

[*] See for example Sen (1980, 1985a, 1985b, 1987, 1992, 1993)
[†] See Tsui (1999, 2002), Atkinson (2003), Bourguignon and Chakravarty (2003), Duclos, Sahn, and Younger (2006), Thorbecke (2008), and Kakwani and Silber (2008b), among others. For discussions of unidimensional poverty measurement, see Sen (1976), Blackorby and Donaldson (1980), Clark, Hemming and Ulph (1981), Chakravarty (1983), Foster, Greer and Thorbecke (1984), Atkinson (1987), Ravallion (1996), Sen (1997), and the surveys of Foster and Sen (1997), Zheng (1997), and Foster (2006).

the 'intersection' method, which requires a person to be deprived in all dimensions before being identified as poor. This is often considered too constricting, and generally produces untenably low estimates of poverty. Empirical assessments of multidimensional poverty will require a satisfactory solution to the identification question, and although the problems with existing approaches are widely acknowledged, an acceptable alternative has yet to be found. In what follows we provide a first step towards addressing this issue.

This paper introduces an intuitive approach to identifying the poor that uses two forms of cutoffs. The first is the traditional dimension-specific poverty line or cutoff, which identifies whether a person is deprived with respect to that dimension. The second delineates how widely deprived a person must be in order to be considered poor.[*] Our benchmark procedure uses a counting methodology[†], in which the second cutoff is a minimum number of dimensions of deprivation.

The 'dual cutoff' method of identification naturally suggests an approach to aggregation that is likewise sensitive to the range of deprivations a poor person experiences. We derive a new class of 'dimension-adjusted' multidimensional poverty measures based on the traditional *FGT* measures of poverty. The new methodology satisfies an array of desirable axioms including 'decomposability', a property that facilitates targeting, and a new requirement of 'dimensional monotonicity', by which an expansion in the range of deprivations experienced by a poor person is reflected in the overall level of poverty.

Many capabilities can only be represented by ordinal data, yet virtually all existing multidimensional poverty measures require cardinal data. The one exception is the multidimensional headcount ratio, which violates dimensional monotonicity. In contrast, our dimension-adjusted headcount ratio works with ordinal data, respects dimensional monotonicity, and can be undergirded by a neat axiomatic structure on individual poverty functions based on the counting result of Pattanaik and Xu (1990) in the literature on measuring freedom.

In some circumstances we may have additional information that allows us to regard certain dimensions as meriting greater relative weight than others. In such cases our

[*] In this paper we will use the term 'deprived' to indicate that a person's achievement in a given dimension falls below the cutoff. If a person meets the multidimensional identification criterion, we refer to them as 'poor', and their condition as 'poverty'.

[†] Our approach was motivated in part by Atkinson (2003), who explored the relationship between counting and social welfare methods at the aggregation step.

identification procedure and the associated additive poverty measures can be easily generalised from equal weights across the dimensions to general weights. We do this in our final methodological section.

An important consideration in developing a new methodology for measuring poverty is that it can be employed using real data to obtain meaningful results. To show this is true for our methodology, we provide illustrative examples using data from Indonesia and the US. In sum, the methodology we propose is intuitive, satisfies useful properties, and can be applied to good effect with real world data.

The structure of the paper is as follows. We begin with a brief introduction to unidimensional poverty measurement as it provides a foundation for our departure into multidimensional space. We present some basic definitions and notation for multidimensional poverty, and then introduce our dual cutoff identification strategy. The adjusted *FGT* family of poverty measures is introduced, and we provide a list of axioms that are satisfied by the methodology. The next section discusses the case where the data are ordinal variables, and observes that one of our measures, the dimension-adjusted headcount ratio, works well in this context. We present a theorem that characterizes both the identification method and the aggregate measure in this environment, using the counting approach of Pattanaik and Xu. We show how to extend our methods to allow for general weights, and supply two informative illustrations using data from Indonesia and the US. A final section offers closing observations.

2. UNIDIMENSIONAL MEASUREMENT

Poverty measurement can be broken down into two distinct steps: ‘identification’ which defines the criteria for distinguishing poor persons from the non-poor, and ‘aggregation’ by which data on poor persons are brought together into an overall indicator of poverty (Sen 1976). Identification typically makes use of an income cutoff called the *poverty line* and evaluates whether an individual’s income achieves this level. Aggregation is typically accomplished by selecting a *poverty index* or *measure*.

The simplest and most widely used poverty measure is the *headcount ratio*, which is the percentage of a given population that is poor. A second index, the (*per capita*) *poverty gap*, identifies the aggregate by which the poor fall short of the poverty line income, measured in poverty line units and averaged across the population. Both indices can be seen as a population average, where the non-poor are assigned a value of ‘0’. The headcount ratio assigns a value of ‘1’ to all poor persons, while the poverty gap assigns the *normalised shortfall* (the difference between their income and the poverty line, divided by the poverty line itself) before taking the population average. Unlike the headcount ratio, the poverty gap is sensitive to income decrements among the poor and registers an increase when the shortfall of a poor person rises.

A third method of aggregation suggested by Foster, Greer and Thorbecke (1984) proceeds as above for each person who is not poor, but now transforms the normalised shortfalls of the poor by raising them to a nonnegative power α to obtain the associated P_α or *FGT* measure. This approach includes both of the foregoing measures: if $\alpha = 0$, the headcount ratio is obtained; if $\alpha = 1$, we have the poverty gap measure. The value $\alpha = 2$ results in the *FGT* index P_2, which is a simple average of the squared normalized shortfalls across society. Squaring the normalised gaps diminishes the relative importance of smaller shortfalls and augments the effect of larger ones. Consequently P_2 emphasises the conditions of the poorest poor in society.

Every poverty index has different insights and oversights, and one way of illuminating them is to identify the properties or axioms the index satisfies. Each property captures a basic desideratum for an aggregation method, and usually defines a form of stylised change in the distribution that should impact the poverty measure in a prescribed way. As is well-known, the *FGT* measures satisfy a broad array of properties, including *symmetry*, *replication invariance*, *subgroup consistency* and *decomposability*; specific members satisfy *monotonicity* ($\alpha > 0$) and the *transfer* axiom ($\alpha > 1$). We will build on this family of measures when we develop our multidimensional methodology below.

3. NOTATION

Moving from the unidimensional to a multidimensional poverty framework raises a set of significant questions: *(i)* which are the dimensions, and indicators, of interest? *(ii)* where should cutoffs be set for each dimension? *(iii)* how should dimensions be weighted? *(iv)* how can we identify the multidimensionally poor? *(v)* what multidimensional poverty measure(s) should be used? *(vi)* which measures can accommodate ordinal data? *(vii)* should multidimensional poverty measures reflect interactions between dimensions, and if so, how? Issues *(i)* through *(iii)* have been substantially discussed in the literature;[*] in what follows we will assume that

[*] On the selection of capabilities or dimensions see Sen (1992a, 1993, 2004a, 2004b), Alkire (2002, 2008), Atkinson *et al* (2002), Qizilbash (2002), Nussbaum (2003), Robeyns (2005), and Ranis, Stewart

appropriate judgements have been made. The present paper is concerned with questions *(iv) – (vi)*: identification in a multidimensional setting, the construction of an aggregate measure, and how to measure poverty when data are only ordinally significant. Issue *(vii)* is still an open question,[*] and in the interest of making progress on *(iv)* through *(vi)*, we adopt a neutral position in the present paper. The desirability of accounting for 'complements' and 'substitutes' directly in poverty measures is discussed in section 10 below.

Let n represent the number of persons and let $d \geq 2$ be the number of dimensions under consideration. Let $y = [y_{ij}]$ denote the $n \times d$ matrix of achievements, where the typical entry $y_{ij} \geq 0$ is the achievement of individual $i = 1,2,\ldots, n$ in dimension $j = 1,2,\ldots, d$. Each row vector y_i lists individual i's achievements, while each column vector y_{*j} gives the distribution of dimension j achievements across the set of individuals. In what follows we assume that d is fixed and given, while n is allowed to range across all positive integers; this allows poverty comparisons to be made across populations of different sizes. Thus the domain of matrices under consideration is given by $Y = \{y \in R_+^{nd} : n \geq 1\}$.[†] Let $z_j > 0$ denote the cutoff below which a person is considered to be deprived in dimension j, and let z be the row vector of dimension-specific cutoffs. For any vector or matrix v, the expression $|v|$ denotes the sum of all of its elements, while $\mu(v)$ represents the mean of v, or $|v|$ divided by the total number of elements in v.

A methodology $\mathcal{M}$ for measuring multidimensional poverty is made up of an identification method and an aggregate measure. Following Bourguignon and Chakravarty (2003) we represent the former using an *identification function* $\rho: R_+^d \times R_{++}^d \rightarrow \{0,1\}$, which maps from person i's achievement vector $y_i \in R_+^d$ and cutoff vector z in R_{++}^d to an indicator variable in such a way that $\rho(y_i; z) = 1$ if person i is poor and

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and Samman (2006), and Thorbecke (2008). On the setting of poverty lines see Sen (1981), Foster and Sen (1997), Foster (1998), Ravallion (1998); on fuzzy set methods see Cerioli and Zani (1990), Chiappero-Martinetti (1994, 1996, 2000, 2008), Cheli and Lemmi (1995), Balestrino (1998), Qizilbash (2003), and Betti *et al* (2008). Techniques for applying weights across dimensions include arbitrary weights, statistical weights (e.g. factor analysis or multiple correspondence analysis), survey-based weights, normative weights or a combination of these techniques. See Sen (1980, 1985, 1987, 1992), Brandolini and D'Alessio (1998), Alkire and Clark (2008) and Lugo and Decanq (2008).

[*] See Tsui (2002), Bourguignon and Chakravarty (2003, p. 27-8), Duclos, Sahn and Younger (2006), Kakwani and Silber (2008a, 2008b), Maasoumi and Lugo (2008), Thorbecke (2008) among others.

[†] For concreteness, we assume that individual achievements can be any non-negative real; our approach can easily accommodate larger or smaller domains when appropriate.

$\rho(y_i; z) = 0$ if person i is not poor.[*] Applying ρ to each individual achievement vector in y yields the set $Z \subseteq \{1,\ldots, n\}$ of persons who are poor in y given z. The aggregation step then takes ρ as given and associates with the matrix y and the cutoff vector z an overall level $M(y; z)$ of multidimensional poverty. The resulting functional relationship $M: Y \times R^d_{++} \rightarrow R$ is called an *index*, or *measure*, of multidimensional poverty. This paper presents a new methodology $\mathcal{M} = (\rho, M)$ for measuring multidimensional poverty, explores its properties and provides illustrative examples.

In what follows, it will prove useful to express the data in terms of deprivations rather than achievements. For any given y, let $g^0 = [g^0_{ij}]$ denote the 0-1 *matrix of deprivations* associated with y, whose typical element g^0_{ij} is defined by $g^0_{ij} = 1$ when $y_{ij} < z_j$, while $g^0_{ij} = 0$ otherwise. Clearly, g^0 is an $n \times d$ matrix whose ij^{th} entry is 1 when person i is deprived in the j^{th} dimension, and 0 when the person is not. The i^{th} row vector of g^0, denoted g^0_i, is person i's *deprivation vector*. From the matrix g^0 we can construct a column vector c of *deprivation counts*, whose i^{th} entry $c_i = |g^0_i|$ represents the number of deprivations suffered by person i. The vector c will be especially helpful in describing our method of identification. Notice that even when the variables in y are only ordinally significant, g^0 and c are still well defined.[†]

If the variables in y are cardinal, the associated matrix of (normalised) gaps or shortfalls can provide additional information for poverty evaluation. For any y, let g^1 be the matrix of *normalised gaps*, where the typical element is defined by $g^1_{ij} = (z_j - y_{ij})/z_j$ whenever $y_{ij} < z_j$, while $g^1_{ij} = 0$ otherwise. Clearly, g^1 is an $n \times d$ matrix whose entries are nonnegative numbers less than or equal to 1, with g^1_{ij} being a measure of the extent to which that person i is deprived in dimension j. In general, for any $\alpha > 0$, define the matrix g^α by raising each entry of g^1 to the power α; e.g. when $\alpha = 2$, the entry is $g^2_{ij} = (g^1_{ij})^2$. This notation will be useful below in defining our generalisation of the *FGT* measures to the multidimensional environment.

[*] Note that this representation assumes that the underlying identification method is individualistic (in that i's poverty status depends on y_j) and symmetric (in that it uses the same criterion for all persons). It would be interesting to explore a more general identification function which abstracts from these assumptions.

[†] In other words, g^0 and c are identical for all monotonic transformations of y_{ij} and z_j. See section 7 below.

4. IDENTIFYING THE POOR

Who is poor and who is not? A reasonable starting place is to compare each individual's achievements against the respective dimension-specific cutoffs, and we follow that general strategy here.* But dimension specific cutoffs alone do not suffice to identify who is poor; we must consider additional criteria that look *across* dimensions to arrive at a complete specification of identification method. We now examine some potential candidates for $\rho(y_i; z)$.

The 'unidimensional' method aggregates all achievements into a single cardinal variable of 'well-being' or 'income' and uses an aggregate cutoff to determine who is poor. So, for example, if y_i is a vector of commodities with market price vector p, one might define $\rho_p(y_i; z) = 1$ whenever $py_i < pz$, and $\rho_p(y_i; z) = 0$ otherwise. In this case, a person is poor if the monetary value of the achievement bundle is below the cost of the target bundle z. More generally, one might invoke an aggregator function u such that $\rho_u(y_i; z) = 1$ whenever $u(y_i) < u(z)$, and $\rho_u(y_i; z) = 0$ otherwise. However, the unidimensional form of identification entails a host of assumptions that restrict its applicability in practice, and its desirability in principle.† From the perspective of the capability approach, a key conceptual drawback of viewing multidimensional poverty through a unidimensional lens is the loss of information on dimension-specific shortfalls: indeed, aggregation before identification converts dimensional achievements into one another without regard to dimension-specific cutoffs. If, as argued above, dimensions are independently valued and dimensional deprivations are inherently undesirable, then there are good reasons to look beyond a unidimensional approach to identification methods that focus on dimensional shortfalls.

The most commonly used identification criterion of this type is called the *union* method of identification. In this approach, a person i is said to be multidimensionally poor if there is at least one dimension in which the person is deprived (i.e., $\rho(y_i; z) = 1$ if

* See, for example, Bourguignon and Chakravarty (2003, p 27-8) who contend that "a multidimensional approach to poverty defines poverty as a shortfall from a threshold on each dimension of an individual's well being."

† One common assumption is that prices exist and are adequate, normative weights for the dimensions; however, as noted by Tsui (2002) this assumption is questionable. Prices may be adjusted to reflect externalities, but exchange values do not and 'indeed cannot give … *interpersonal comparisons* of welfare or advantage' (Sen (1997, p. 208)). Pradhan and Ravallion (1996) derive subjective poverty lines in place of prices, but cannot do so for all attributes. Additional problems can arise when markets are missing or imperfect (Bourguignon & Chakravarty (2003), Tsui (2002)). Also, empirical evidence shows that income may not be translated into basic needs (Ruggeri-Laderchi, Saith and Stewart (2003), Sen (1980)). Aggregating across dimensions for purposes of identification also entails strong assumptions regarding cardinality, which are impractical when data are ordinal (Sen (1997)).

and only if $c_i \geq 1$). If sufficiency in every dimension were truly essential for avoiding poverty, this approach would be quite intuitive and straightforward to apply. However, it might also include persons whom many would not consider to be poor. For example, deprivation in certain single dimensions (such as health or education) may be reflective of something other than poverty. Moreover, a union based poverty methodology may not be helpful for distinguishing and targeting the poorest of the poor, especially when the number of dimensions is large. For these reasons the union method, though commonly used – for example (implicitly) in well-known measures such as the Human Poverty Index (*HPI*) – is not unambiguously acceptable.

The other identification method of this type is the *intersection* approach, which identifies person *i* as being poor only if the person is deprived in all dimensions (i.e., $\rho(y_i; z) = 1$ if and only if $c_i = d$). This criterion would accurately identify the poor if sufficiency in any single dimension were enough to prevent poverty; indeed, it successfully identifies as poor a group of especially deprived persons. However, it inevitably misses many persons who are experiencing extensive, but not universal, deprivation (for example, a destitute person who happens to be healthy). Moreover, it succeeds in identifying only a narrow slice of the population that shrinks as the number of dimensions increases – and disregards the rest. This creates a different tension, that of considering persons to be non-poor who evidently suffer considerable deprivation.

A natural alternative is to use an intermediate cutoff level for c_i that lies somewhere between the two extremes of 1 and d. For $k = 1,\ldots, d$, let ρ_k be the identification method defined by $\rho_k(y_i; z) = 1$ whenever $c_i \geq k$, and $\rho_k(y_i; z) = 0$ whenever $c_i < k$. In other words, ρ_k identifies person i as poor when the number of dimensions in which i is deprived is at least k; otherwise, if the number of deprived dimensions falls below the cutoff k, then i is not poor according to ρ_k. Since ρ_k is dependent on both the *within dimension* cutoffs z_j and the *across dimension* cutoff k, we will refer to ρ_k as the *dual cutoff* method of identification.[*] Notice that ρ_k includes the union and intersection methods as special cases where $k = 1$ and $k = d$.

Similar methods of identification can be found in the literature, albeit with different motivations. For example, Mack and Lansley *Poor Britain* (1985) identified people as poor if they were poor in 3 or more out of 26 deprivations. The UNICEF *Child Poverty*

[*] We do not provide an algorithm for selecting k here; instead, repeated application and reasoned evaluation will likely lead to a range of plausible values for k. A single value can then be selected for the main analysis and alternative values used to check robustness.

Report 2003 identified any child who was poor with respect to two or more deprivations as being in extreme poverty (Gordon, *et al.*, 2003). However, as a general methodology for identifying the poor, the dual cutoff approach has not been explicitly formulated in the literature, nor have its implications for multidimensional poverty measurement been explored.[*]

The dual cutoff method has a number of characteristics that deserve mention. First, it is 'poverty focused' in that an increase in an achievement level y_{ij} of a non-poor person leaves its value unchanged. Second, it is 'deprivation focused' in that an increase in any non-deprived achievement $y_{ij} \geq z_j$ leaves the value of the identification function unchanged; in words, a person's poverty status is not affected by changes in the levels of non-deprived achievements. This latter property separates ρ_k from the unidimensional method ρ_u, which allows a higher level of one achievement to compensate for other dimensional deprivations in deciding who is poor or non-poor. Finally, the dual cutoff identification method can be meaningfully used with ordinal data, since a person's poverty status is unchanged when a monotonic transformation is applied to an achievement level and its associated cutoff.[†] This clearly rules out ρ_u, which aggregates dimensions *before* identifying the poor, and thus can be altered by monotonic transformations.

In the next section, we introduce multidimensional poverty measures based on the Foster Greer Thorbecke *(FGT)* class that use the ρ_k identification method and its associated set $Z_k = \{i : \rho_k(y_i; z) = 1\}$ of poor people. Accordingly, we will make use of some additional notation that censors the data of non-poor persons. Let $g^0(k)$ be the matrix obtained from g^0 by replacing the i^{th} row with a vector of zeros whenever $\rho_k(y_i; z) = 0$, and define $g^\alpha(k)$ analogously for $\alpha > 0$. The typical entry of $g^\alpha(k)$ is thus given by $g_{ij}^\alpha(k) = g_{ij}^\alpha$ for i satisfying $c_i \geq k$, while $g_{ij}^\alpha = 0$ for i with $c_i < k$ or, equivalently, by $g_{ij}^\alpha(k) = g_{ij}^\alpha \rho_k(y_i; z)$ As the cutoff k rises from 1 to d, the number of nonzero entries in the associated matrix $g^\alpha(k)$ falls, reflecting the progressive censoring of data from persons who are not meeting the dimensional poverty requirement presented by ρ_k. It is clear that the union specification $k = 1$ does not alter the original matrix at all; consequently,

[*] An analogous approach has been used in the measurement of chronic poverty, with duration in that context corresponding to breadth in the present case. See Foster (2007).

[†] In other words, $\rho_k(y_i; z) = \rho_k(y_i'; z')$ where for each $j = 1,\ldots,d$ we have $y'_{ij} = f_j(y_{ij})$ and $z_j' = f_j(z_j)$ for some increasing function f_j. It would be interesting to characterize the identification methods ρ satisfying the above three properties.

$g^{\alpha}(1) = g^{\alpha}$. The intersection specification $k = d$ removes the data of any person who is not deprived in all d dimensions; in other words, when the matrix $g^{\alpha}(d)$ is used, a person deprived in just a single dimension is indistinguishable from a person deprived in d–1 dimensions. When $k = 2,\ldots, d–1$, the dual cutoff approach provides an intermediate option between the union and intersection methods as reflected in the matrix $g^{\alpha}(k)$.

5. MEASURING POVERTY

We are searching for a multidimensional poverty measure $M(y; z)$ to be used with the dual cutoff identification approach. A natural place to begin is with the percentage of the population that is poor. The headcount ratio $H = H(y; z)$ is defined by $H = q/n$, where $q = q(y; z)$ is number of persons in the set Z_k, and hence the number of the poor identified using the dual cutoff approach. This is entirely analogous to the income headcount ratio and inherits the virtue of being easy to compute and understand, and the weakness of being a crude, or partial, index of poverty.[*] Notice, though, that an additional problem emerges in the multidimensional setting. If a poor person becomes deprived in a dimension in which that person had previously not been deprived, H remains unchanged. This violates what we will call 'dimensional monotonicity' which is defined rigorously below. Intuitively speaking, if poor person i becomes newly deprived in an additional dimension, then overall poverty should increase.

To reflect this concern, we can include additional information on the breadth of deprivation experienced by the poor. Let k be an integer between 1 and d. We define the *censored vector of deprivation counts* c(k) as follows: If $c_i \geq k$, then $c_i(k) = c_i$, or person i's deprivation count; if $c_i < k$, then $c_i(k) = 0$. Notice that $c_i(k)/d$ represents the share of possible deprivations experienced by a poor person i, and hence the *average deprivation share* across the poor is given by $A = |c(k)|/(qd)$. This partial index conveys relevant information about multidimensional poverty, namely, the fraction of possible dimensions d in which the average poor person endures deprivation. Consider the following multidimensional poverty measure $M_0(y;z)$ which combines information on the prevalence of poverty and the average extent of a poor person's deprivation.

DEFINITION 1: The *(dimension) adjusted headcount ratio* is given by $M_0 = HA$.

[*] A partial index provides information on only one aspect of poverty. See Foster and Sen (1997).

As a simple product of the two partial indices H and A, the measure M_0 is sensitive to the frequency and the breadth of multidimensional poverty. In particular, it clearly satisfies dimensional monotonicity, since if a poor person becomes deprived in an additional dimension, then A rises and so does M_0. The adjusted headcount ratio can be used with purely ordinal data, which arises frequently in multidimensional approaches based on capabilities. This important characteristic of the measure will be discussed at some length in a separate section below. Note that M_0 can be defined as $M_0 = \mu(g^0(k))$, or the mean of the censored deprivation matrix $g^0(k)$. In words, the adjusted headcount ratio is the total number of deprivations experienced by the poor, or $|c(k)| = |g^0(k)|$, divided by the maximum number of deprivations that could possibly be experienced by all people, or nd.

The adjusted headcount ratio is based on a dichotomisation of the data into deprived and non-deprived dimensions, and so it does not make use of dimension specific information on the depth of deprivation. Consequently it will not satisfy the traditional monotonicity requirement that poverty should increase as a poor person becomes more deprived in any given dimension. To develop a measure that is sensitive to the depth of deprivation, we return to the matrix g^1 of normalised gaps and its associated censored version $g^1(k)$. Let G be the *average poverty gap* across all instances in which poor persons are deprived, given by $G = |g^1(k)|/|g^0(k)|$. Consider the following multidimensional poverty measure $M_1(y;\ z)$ which combines information on the prevalence of poverty, the average range of deprivations and the average depth of deprivations when the poor are deprived.

DEFINITION 2: The *(dimension) adjusted poverty gap* is given by $M_1 = HAG$.

The adjusted poverty gap is thus the product of the adjusted headcount ratio M_0 and the average poverty gap G. It is easily shown that $M_1 = \mu(g^1(k))$; in words, the adjusted poverty gap is the sum of the normalised gaps of the poor, or $|g^1(k)|$ divided by the highest possible sum of normalised gaps, or nd. If the deprivation of a poor person deepens in any dimension, then the respective $g^1_{ij}(k)$ will rise and hence so will M_1. Consequently, M_1 satisfies monotonicity. However, it is also true that the increase in a deprivation has the same impact no matter whether the person is very slightly deprived or acutely deprived in that dimension. One might argue that the impact should be larger in the latter case.

Consider the matrix g^2 of squared normalised shortfalls and its censored version $g^2_{ij}(k)$. These matrices provide information on the severity of deprivations as measured by the square of the normalised shortfalls, with the censored matrix $g^2(k)$ including only the data on the poor. Rather than using the matrix $g^1(k)$ to supplement the information of M_0 (as was done in M_1), we can use the matrix $g^2(k)$ which suppresses the smaller gaps and emphasises the larger ones. The *average severity* of deprivations, across all instances in which poor persons are deprived, is given by $S = |g^2(k)|/|g^0(k)|$. The following multidimensional poverty measure $M_2(y;z)$ combines information on the prevalence of poverty and the range and severity of deprivations.

DEFINITION 3: The *(dimension) adjusted P_2 measure* is given by $M_2 = HAS$.

M_2 is thus the product of the adjusted headcount ratio M_0 and the average severity index S; it can also be expressed as $M_2 = \mu(g^2(k))$, the mean of the matrix $g^2(k)$, which in words is the sum of the squared normalised gaps of the poor, or $|g^2(k)|$, divided by the highest possible sum of the squared normalised gaps, or nd. For a given sized increase in deprivation, the measure registers a greater impact the larger the initial level of deprivation. It satisfies a 'transfer' property (as noted below), and is sensitive to the inequality with which deprivations are distributed among the poor, and not just their average level. Indeed, $M_2 = (M_1)^2 + V$, where V is the variance among all normalised gaps.*

It is straightforward to generalise M_0, M_1, and M_2, to a class M_α of multidimensional poverty measures associated with the unidimensional *FGT* class developed by Foster Greer and Thorbecke (1984). For every $\alpha \geq 0$, let g^α be the matrix whose entries are α powers of the normalised gaps, and let $g^\alpha(k)$ be the associated censored matrix.† Consider the following class of measures.

DEFINITION 4: The *(dimension) adjusted FGT measures*, denoted $M_\alpha(y;z)$, are given by $M_\alpha = \mu(g^\alpha(k))$ for $\alpha \geq 0$.

* In other words, $V = \Sigma_i\Sigma_j(\mu(g^1) - g^1_{ij})^2/(nd)$. The formula can also be expressed as $M_2 = (M_1)^2[1 + C^2]$, where $C^2 = V/(\mu(g^1))^2$ is the squared coefficient of variation inequality measure. This is analogous to a well-known formula for the *FGT* measure P_2.

† Technically speaking, this definition applies only for $\alpha > 0$. The matrix g^0 (or $g^0(k)$) can be obtained as the limit of g^α (respectively, $g^\alpha(k)$) as α tends to 0.

In other words, M_α is the sum of the α powers of the normalised gaps of the poor, or $|g^\alpha(k)|$, divided by the highest possible value for this sum, or nd. The poverty measure M_α ranges in value from 0 to 1. We now turn to a discussion of the properties satisfied by M_α and H.

6. PROPERTIES

The traditional approach to constructing properties for multidimensional poverty measures has been to alter their unidimensional counterparts in natural ways.[*] However in the multidimensional context, the identification step is no longer elementary, and properties must be viewed as joint restrictions on the identification method ρ and aggregate measure M and, thus, on the overall methodology $\mathcal{M}$. Some properties (such as 'symmetry' below) only use ρ in finding poverty levels. Others (such as 'poverty focus') make explicit use of ρ to restrict consideration to certain data matrices or changes covered by the axiom. In the following discussion, we will assume that a specific ρ has been selected and will use the statement 'M satisfies axiom A' as shorthand for '(ρ, M) satisfies axiom A'. In particular, ρ_k will be the identification method used whenever M_α or H is being discussed.[†]

A key property satisfied by M_α and H is 'decomposability' which requires overall poverty to be the weighted average of subgroup poverty levels, where weights are subgroup population shares. In symbols, let x and y be two data matrices and let (x,y) be the matrix obtained by merging the two; let $n(x)$ be the number of persons in x (and similarly for $n(y)$ and $n(x,y)$).

DECOMPOSABILITY: For any two data matrices x and y we have

$$M(x,y;z) = \frac{n(x)}{n(x,y)} M(x;z) + \frac{n(y)}{n(x,y)} M(y;z).$$

Repeated application of this property shows that the decomposition holds for any number of subgroups, making this an extremely useful property for generating profiles of poverty and targeting high poverty populations.[‡] If we apply a decomposable

[*] Tsui (1999, 2002), Atkinson (2003), Bourguignon & Chakravarty (2003), Duclos Sahn and Younger (2006), and Kakwani and Silber (2008b).

[†] Note that the identification method ρ_k could also be used with other existing multidimensional poverty measures such as Tsui (2002), Bourguignon and Chakravarty (2003), or Massoumi and Lugo (2008).

[‡] Any decomposable measure also satisfies 'subgroup consistency' which requires overall poverty to increase when poverty rises in the first subgroup and does not fall in the second (given fixed population

measure to a *replication* x of y, which has the form $x = (y,y,\ldots,y)$, it follows that x has the same poverty level as y. The following basic property is thus satisfied by M_α and H.

REPLICATION INVARIANCE: If x is obtained from y by a replication, then $M(x; z) = M(y; z)$.

This property ensures that poverty is measured relative to the population size, so as to allow meaningful comparisons across different sized populations. Now let x be obtained from y by a *permutation*, by which it is meant that $x = \Pi y$, where Π is some $n \times n$ permutation matrix.* This has the effect of reshuffling the vectors of achievements across people. It is immediately clear from the definitions of M_α and H that they satisfy the following property:

SYMMETRY: If x is obtained from y by a permutation, then $M(x; z) = M(y; z)$.

According to symmetry, if two or more persons switch achievements, measured poverty is unaffected. This ensures that the measure does not place greater weight on any person or group of persons.

The traditional focus axiom requires a poverty measure to be independent of the data of the non-poor, which in the unidimensional or income poverty case is simply all incomes at or above the single poverty line.† In a multidimensional setting, a non-poor person can be deprived in several dimensions while a poor person may well exceed several of the deprivation cutoffs. M_α and H satisfy two forms of the focus axiom, one concerning the poor, and the other pertaining to deprived dimensions. We say that x is obtained from y by a *simple increment* if $x_{ij} > y_{ij}$ for some pair $(i, j) = (i', j')$ and $x_{ij} = y_{ij}$ for every other pair $(i, j) \neq (i', j')$. We say it is a simple increment *among the non-poor* if i' is not in Z for y (whether i' is deprived or not in j'); it is a simple increment *among the nondeprived* if $y_{ij} \geq z_j$ for $(i, j) = (i', j')$, whether or not i' happens to be poor.

sizes). As discussed in Foster, Greer and Thorbecke (1984) and Foster and Sen (1997), it is this property that allows the coordination of local and national poverty alleviation policies.

* A permutation matrix Π is square matrix with a single '1' in each row and each column, and the rest '0's.

† An alternative definition considers persons on or below the cutoff to be poor.

POVERTY FOCUS: If x is obtained from y by a simple increment among the non-poor, then $M(x; z) = M(y; z)$.

DEPRIVATION FOCUS: If x is obtained from y by a simple increment among the nondeprived, then $M(x; z) = M(y; z)$.

In the poverty focus axiom, the set Z of the poor is identified using ρ, and M is required to be unchanged when anyone outside of Z experiences a simple increment. This is a basic requirement that ensures that M measures poverty in a way that is consistent with the identification method ρ. In the case of M_α and H, the poor are identified using ρ_k and the achievements of the non-poor are censored prior to aggregation. Hence, they satisfy the poverty focus axiom. In the deprivation focus axiom, the simple increment is defined independently of the particular identification method employed and is applicable to all nondeprived entries in y – poor and non-poor alike. For the measures M_α and H, a simple increment to a nondeprived entry leaves $g^\alpha(k)$ unchanged, and hence they satisfy the deprivation focus axiom as well.

It is possible for a multidimensional poverty methodology to follow the poverty focus axiom without satisfying the deprivation focus axiom. Consider, for example, a unidimensional approach that, say, adds the dimensions to create an income variable, identifies the poor using an aggregate cutoff and employs a standard income poverty measure. Given the assumed tradeoffs across dimensions, it is possible for a poor person to be lifted out of poverty as a result of an increment in a nondeprived dimension, thus lowering the measured level of poverty and violating deprivation focus. Conversely, the deprivation focus axiom may be satisfied without accepting the poverty focus axiom: suppose the average gap $\mu(g^1)$ over all deprivations (poor or non-poor) is taken to be the measure and yet take an intersection approach to identification is used.[*]

The next set of properties ensures that a multidimensional poverty measure has the proper orientation. Consider the following extensions to the definition of a simple increment: We say that x is obtained from y by a *deprived increment among the poor* if in addition to being a simple increment we have $z_{j'} > y_{i'j'}$ for $i' \in Z$; it is a *dimensional*

[*] The two forms of focus axioms are related in certain cases. When union identification is used, it can be shown that the deprivation focus axiom implies the poverty focus axiom; alternatively, when an intersection approach is used, the poverty focus axiom implies the deprivation version. Bourguignon and Chakravarty (2003), for example, assume the deprivation focus axiom (their 'strong focus axiom') along with union identification, and so their methodology automatically satisfies the poverty focus axiom.

increment among the poor if it satisfies $x_{i'j'} \geq z_{j'} > y_{i'j'}$ for $i' \in Z$. In other words, a deprived increment among the poor improves a deprived achievement of a poor person, while a dimensional increment among the poor completely removes the deprivation. Consider the following properties.

WEAK MONOTONICITY: If x is obtained from y by a simple increment, then $M(x; z) \leq M(y; z)$.

MONOTONICITY: M satisfies weak monotonicity and the following: if x is obtained from y by a deprived increment among the poor then $M(x; z) < M(y; z)$.

DIMENSIONAL MONOTONICITY: If x is obtained from y by a dimensional increment among the poor, then $M(x; z) < M(y; z)$.

Weak monotonicity ensures that poverty does not increase when there is an unambiguous improvement in achievements. Monotonicity additionally requires poverty to fall if the improvement occurs in a deprived dimension of a poor person. Dimensional monotonicity specifies that poverty should fall when the improvement removes the deprivation entirely; it is clearly implied by monotonicity. Every M_α and H satisfy weak monotonicity; every M_α (and not H) satisfies dimensional monotonicity; and every M_α measure with $\alpha > 0$ satisfies monotonicity, while H and M_0 do not.

The weak monotonicity and focus axioms ensure that a measure M achieves its highest value at x^0 in which all achievements are 0 (and hence each person is maximally deprived), while it achieves its lowest value at any x^z in which all achievements reach or exceed the respective deprivation cutoffs given in z (and hence no one is deprived). ‘Nontriviality’ ensures that these maximum and minimum values are distinct, while ‘normalisation’ goes further and assigns a value of 1 to x^0 and a value of 0 to each x^z. Both are satisfied by every member of the M_α class and H.

NONTRIVIALITY: M achieves at least two distinct values.

NORMALISATION: M achieves a minimum value of 0 and a maximum value of 1.

For any multidimensional poverty measure satisfying monotonicity, one can explore whether the measure is also sensitive to inequality among the poor. The simplest notion of this sort is based on an 'averaging' of the achievement vectors of two poor persons i and i', in which person i receives $\lambda > 0$ of the first vector and $1-\lambda > 0$ of the second with the shares reversed for person i'. Following Kolm (1977) these d many 'progressive transfers' between the poor represent an unambiguous decrease in inequality, which some would argue should be reflected in a lower or equal value of multidimensional poverty. In general, we say that x is obtained from y by an *averaging of achievements among the poor* if $x = By$ for some $n \times n$ bistochastic matrix[*] B satisfying $b_{ii} = 1$ for every non-poor person i in y. Note that the requirement $b_{ii} = 1$ ensures that all the non-poor columns in y are unaltered in x, while the fact that B is bistochastic ensures that the poor columns in x are obtained as a convex combination of the poor columns in y, and hence inequality has fallen or remained the same. Consider the following property.

WEAK TRANSFER: If x is obtained from y by an averaging of achievements among the poor, then $M(x; z) \leq M(y; z)$.

This axiom ensures that an averaging of achievements among the poor generates a poverty level that is less than or equal to the original poverty level.[†]

We can show that M_α satisfies the weak transfer axiom for $\alpha \geq 1$. Indeed, let x be obtained from y by an averaging of achievements among the poor. Then where q is the number of poor persons in y, let y' be the matrix obtained from y by replacing each of the n-q non-poor rows of y with the vector z. Similarly, let x' be the matrix obtained from x by replacing the same n-q rows with z. Clearly $M_\alpha(y; z) = M_\alpha(y'; z)$ and $M_\alpha(x; z) = M_\alpha(x'; z)$. For any data matrix v, let $g^\alpha(v)$ denote the matrix of α powers of normalised gaps (or shortfalls) associated with v, and notice that $\mu(g^\alpha(v))$ is a convex function of v for $\alpha \geq 1$. Since $x' = By'$ for some bistochastic matrix B, it follows that $\mu(g^\alpha(x')) \leq \mu(g^\alpha(y'))$. But $M_\alpha(y';z) = \mu(g^\alpha(y'))$ by the construction of y', and if the number of poor in x is q, then $M_\alpha(x';z) = \mu(g^\alpha(x'))$ and we would be done. However, it is also possible that the number of poor in x is less than q; in other words the smoothing process has moved

[*] A bistochastic matrix is a nonnegative square matrix having the property that the sum of the elements in each row (or column) is 1.

[†] See Tsui (1999) who calls this property the Poverty Non-increasing Minimal Transfer Axiom.

at least one person from being poor to being non-poor. Then it follows that the associated rows in $g^{\alpha}(x')$ will need to be censored in measuring $M_{\alpha}(x'; z)$, implying that $M_{\alpha}(x'; z) \leq \mu(g^{\alpha}(x'))$. Either way, it follows that $M_{\alpha}(x; z) \leq M_{\alpha}(y; z)$ and hence M_{α} satisfies the weak transfer axiom for $\alpha \geq 1$.

A second notion of sensitivity to inequality can be defined following the work of Atkinson and Bourguignon (1982). The concept is based on a different sort of 'averaging' across two poor persons, whereby one person begins with weakly more of each achievement than a second person, but then switches one or more achievement levels with the second person so that this ranking no longer holds. Motivated by Boland and Proschan (1988), we say x is obtained from y by a *simple rearrangement among the poor* if there are two persons i and i' who are poor in y, such that for each j either $(x_{ij}, x_{i'j}) = (y_{ij}, y_{i'j})$ or $(x_{ij}, x_{i'j}) = (y_{i'j}, y_{ij})$, and for every other person $i'' \neq i, i'$ we have $x_{i''j} = y_{i''j}$. In other words, a simple rearrangement among the poor reallocates the achievements of the two poor persons but leaves the achievements of everyone else unchanged. We say x is obtained from y by an *association decreasing rearrangement among the poor* if, in addition, the achievement vectors of i and i' are comparable by vector dominance in y but are not comparable in x. The following property ensures that reducing inequality in this way generates a poverty level that is less than or equal to the original level.

WEAK REARRANGEMENT: If x is obtained from y by an association decreasing rearrangement among the poor, then $M(x; z) \leq M(y; z)$.

To see that all M_{α} and H satisfy the axiom, notice that the rearrangement does not change the set of the poor nor the collection of achievements among the poor. Hence, both H and M_{α} are unaffected by the rearrangement and just satisfy the axiom.*

In sum then, M_{α} satisfies decomposability, replication invariance, symmetry, poverty and deprivation focus, weak and dimensional monotonicity, nontriviality, normalisation, and weak rearrangement for $\alpha \geq 0$; monotonicity for $\alpha > 0$; and weak transfer for $\alpha \geq 1$. H satisfies all but dimensional monotonicity and monotonicity.

The structure of M_{α} can be utilised to construct the following formulas that are helpful in empirical applications:

* This is called Poverty-Nondecreasing Rearrangement by Tsui (1999).

(1a) $M_\alpha(y; z) = \Sigma_i \, \mu(g_i^\alpha(k))/n$

(1b) $M_\alpha(y; z) = \Sigma_j \, \mu(g_{*j}^\alpha(k))/d$

where $g_i^\alpha(k)$ is the i^{th} row, and $g_{*j}^\alpha(k)$ is the j^{th} column, of the censored matrix $g^\alpha(k)$. In principle, one could apply M_α to the $1 \times d$ 'matrix' containing only the achievement vector y_i of person i, to obtain that person's level of poverty. It turns out that $M_\alpha(y_i; z) = \mu(g_i^\alpha(k))$, and so (1a) becomes $M_\alpha(y; z) = \Sigma_i \, M_\alpha(y_i; z)/n$, or an application of the population decomposition axiom to singleton subgroups. While each achievement vector y_i contains the information necessary to complete the identification step for i, the column vector y_{*j} of j^{th} dimensional achievements does not, since the remaining dimensions are needed to identify the persons who are non-poor and hence the rows that are censored to obtain $g^\alpha(k)$ from g^α. It follows, then, that M_α is not, technically speaking, fully decomposable by dimension. However, once the identification step has been completed and the non-poor rows of g^α have been censored to obtain $g^\alpha(k)$, the above aggregation formula shows that overall poverty is the average of the d many dimensional values $\mu(g_{*j}^\alpha(k))$. Consequently, $(1/d)\mu(g_{*j}^\alpha(k))/M_\alpha(y; z)$ can be interpreted as the post-identification contribution of dimension j to overall multidimensional poverty.*

7. THE ORDINAL CASE

Data that describe capabilities and functionings are often ordinal in nature and collectively may lack a strong basis for making comparisons across dimensions. These aspects present a central challenge to multidimensional poverty measurement based on the capability approach. In this section we consider the problem of ordinal and non-comparable variables and provide a robust solution in the form of the adjusted headcount ratio and related indices.

Certain variables, like income, are commonly taken to be measureable on a ratio scale, which means that they have a natural zero and are unique up to multiplication by a positive constant. Let Λ be the $d \times d$ diagonal matrix having $\lambda_j > 0$ as its j^{th} diagonal element. Matrix multiplying Λ by y and z has the effect of rescaling the dimension j achievements and cutoff by λ_j, which is precisely the transformation allowable for ratio

* Formula (1b) does not provide a full decomposition by dimensions since it takes the identification step as given. The true contribution of a dimension to multidimensional poverty would include its potential impact on identification as well. This is a topic for future discussion.

scale variables. Indeed, it is an easy matter to show that $M_\alpha(y\Lambda;z\Lambda) = M_\alpha(y;\, z)$ and hence the poverty values rendered by the adjusted *FGT* indices are meaningful when achievements are measured as ratio scale variables.[*]

In contrast, if achievements are ordinal variables with no common basis for comparison, then this means that each variable can be independently transformed by an arbitrary increasing function. For $j = 1,\ldots, d$, let f_j: $\mathrm{R}_+ \rightarrow \mathrm{R}_+$ be any strictly increasing function on the nonnegative reals R_+. Let $f(\mathrm{y})$ denote the matrix whose ij^{th} entry is $f_j(y_{ij})$ and let $f(z)$ be the vector whose j^{th} entry is $f_j(z_j)$. Then M_0 has the property that $M_0(f(\mathrm{y});\, f(z)) = M_0(\mathrm{y};\, z)$, and hence the poverty value determined by the adjusted headcount ratio is meaningful even when achievements are ordinal variables.[†] However, for $\alpha > 0$ it is clear that M_α does not share this property, and perhaps more importantly, the underlying *ordering* is not invariant to monotonic transformations of this type. Indeed, for any given $\alpha > 0$ it is easy to construct examples for which $M_\alpha(\mathrm{x};\, z) > M_\alpha(\mathrm{y};\, z)$ and yet $M_\alpha(f(\mathrm{y});\, f(\mathrm{z})) < M_\alpha(f(\mathrm{x});\, f(\mathrm{z}))$. The same critique applies to virtually every multidimensional poverty measure defined in the literature, and so special care must be taken not to use measures whose poverty judgments are meaningless (i.e., reversible under monotonic transformations of the variables) when variables are ordinal. While the headcount ratio *H* does survive this test, it does so at the cost of violating dimensional monotonicity. In contrast, the adjusted headcount ratio provides both meaningful comparisons and favourable axiomatic properties and consequently is recommended when data on achievements are ordinal. In addition, M_0 has an interesting conceptual link to Sen's (1985b, 1985a, 1987, 1992a, 1993) capability framework and the measurement of freedom, which we now pursue in a brief detour.

7.1 *Poverty as Unfreedom*

Sen's capability approach requires a basis for comparing opportunity sets in terms of their levels of 'freedom' or the extent of choice that they allow. Many alternative bases for comparison may be used. Pattanaik and Xu (1990) focus on what Sen calls the intrinsic value of freedom and propose evaluating the freedom of a set in terms of the number of options that are present in the set. A significant literature has further

[*] Notice that each variable is being transformed independently; meaningful comparisons are being obtained without explicitly assuming cross-dimensional comparability of variables.

[†] Note that M_0 can also be applied to certain categorical variables (which do not necessarily admit an ordering across categories), so long as the cutoff category can be compared to all other categories and hence the categories can be dichotomised.

explored and critiqued this theme (Pattanaik and Xu, 1990; Klemisch-Alhert, 1993; Gravel, 1994, 1998; Pattanaik and Xu, 1998, 2000; Bavetta and Del Seta, 2001; Gekker, 2001; Fleurbaey, 2002). In a recent survey of this literature, Foster (2008) used a vector representation of opportunity sets to reinterpret the Pattanaik and Xu result as an additive representation theorem. We will employ Foster's characterisation in the ensuing discussion of M_0 and its identification method.

Let M be a poverty measure satisfying decomposability, weak monotonicity, nontriviality, and a final property of *dichotomisation*, which requires that $M(x; z) = M(y; z)$ for all x and y having the same deprivation matrix g^0. The first three properties are satisfied by all members of M_α; however, M_0 is the only adjusted *FGT* measure that satisfies dichotomisation, and it is this property that ensures that poverty levels and comparisons are meaningful for M_0 when the dimensional variables are ordinal. We will call a measure that satisfies *all four* of these properties a *standard dichotomised measure*.

By decomposability, the structure of M depends entirely on the way that M measures poverty over singleton subgroups; and by dichotomisation, this individual poverty measure can be expressed as a function $F(v)$ of the individual's *deprivation vector* $v = g_i^0$ (which is the i^{th} row vector of 0's and 1's drawn from g^0). In the case of M_0, we have $F(v) = \mu(v(k))$, where $v(k)$ is the censored distribution defined as $v(k) = v$ if $|v| \geq k$ and $v(k)$ is the zero vector of length d if $|v| < k$. We will now explore the possible forms that F can take for standard dichotomised measures. Note that while the definition of M_0 is based on the dual cutoff identification ρ_k, we have not specified the identification method ρ employed by the general index M. Hence a second question of interest is what forms of identification might be consistent with various properties satisfied by M_0.

The individual poverty function F for M_0 has two additional properties of interest. First, it satisfies *anonymity* or the requirement that $F(v) = F(v\Pi)$, where Π is any $d \times d$ permutation matrix.* This property implies that all dimensions are treated symmetrically by the poverty measure. Secondly, it satisfies *semi-independence*, which

* Anonymity is the analogue of Pattanaik and Xu's 'Indifference between No-Choice Situations' (*INS*), when *INS* is taken together with their other assumptions. See Foster (2008).

states that if $v_j = u_j = 1$, and $F(v) \geq F(u)$, then $F(v - e_j) \geq F(u - e_j)$.* Under this assumption, removing the same dimensional deprivation from two deprivation vectors should preserve the (weak) ordering of the two. We have the following result:

THEOREM 1: Let F be the individual poverty function associated with a standard dichotomised poverty measure. F satisfies anonymity and semi-independence if and only if there exists some $k = 1,\ldots, d$ such that for any deprivation vectors v and v' we have: $F(v') \geq F(v)$ if and only if $\mu(v'(k)) \geq \mu(v(k))$.

PROOF: Let $S = \{v \in R^d : v_i = 0$ or $v_i = 1$ for all i$\}$ be the set of all individual deprivation vectors, and let $F: S \rightarrow R$ be an individual poverty function associated with a standard dichotomised poverty measure such that F satisfies anonymity and semi-independence. By anonymity, all vectors v, $v' \in S$ with $|v| = |v'|$ must satisfy $F(v) = F(v')$. In other words, the value of $F(v)$ depends entirely on the number of deprivations in v. Weak monotonicity implies that $F(v) \leq F(v')$ for $|v| \leq |v'|$, and so the value of $F(v)$ is weakly increasing in the number of deprivations in v. By nontriviality and decomposability, it follows that $F(v) > F(0)$ for $|v| = d$. Let k be the lowest deprivation count for which $F(v)$ is strictly above $F(0)$; in other words, $F(v) = F(0)$ for $|v| < k$, and $F(v) > F(0)$ for $|v| \geq k$. Semi-independence ensures that F must be increasing in the deprivation count above k. For suppose that $F(u) = F(u')$ for u, $u' \in S$ with $k \leq |u| < |u'|$. Then by repeated application of anonymity and semi-independence we would have $F(v) = F(v')$ for some v, $v' \in S$ with $|v| < k \leq |v'|$, a contradiction. It follows, then, that $F(v)$ is constant in $|v|$ for $|v| < k$ and increasing in $|v|$ for $k \leq |v|$. Clearly, this is precisely the pattern exhibited by the function $\mu(v(k))$, and hence the proof is complete.

In words, any F satisfying the given assumptions must rank individual deprivation vectors in precisely the same way as the poverty methodology (ρ_k, M_0) for some k. This result is especially powerful, since it characterises not only a poverty index but also the form of identification to be used with it.

The proof of the result follows quite closely the generalisation of Pattanaik and Xu given in Foster (2008). In particular, if full *independence* were required, so that the

* The symbol e_i refers to the i^{th} usual basis vector (0,…,1,…,0) whose only nonzero entry '1' is in the i^{th} coordinate. Note that semi-independence is a weakening of the property of 'Independence' found in Pattanaik and Xu (1990).

conditional in semi-independence were converted to full equivalence, then a direct analogue of the Pattanaik and Xu result would obtain, namely, $F(v') \geq F(v)$ if and only if $\mu(v') \geq \mu(v)$. In this specification, F would make comparisons of individual poverty the same way that the union-identified M_0 does: by counting all deprivations.

While the theorem uniquely identifies the poverty ranking over individual deprivation vectors, it leaves open a multitude of possibilities for the overall index M – one for each specific functional form taken by F. For example, the function $F(v) = \mu(v(k))^2$ ranks individual vectors as before, but generates a different aggregate measure M that places greater emphasis on persons with many deprivations. It would be interesting to explore alternative forms for F and their associated poverty indices – each of which would be applicable to ordinal data.

7.2 *Ordinal and Cardinal Data*

Data available for multidimensional poverty assessment may be ordinal for some dimensions and cardinal for others. Income, for example, is commonly regarded as a cardinal variable while self reported health is generally taken to be purely ordinal.* The mixed case poses no problems for the dual cutoff identification method ρ_k nor for the adjusted headcount measure M_0, which dichotomises all variables before aggregating. However, for M_1 and the other monotonic M_α measures, a tension arises across dimensions: they cannot be applied to ordinal dimensions and yet dichotomisation of cardinal dimensions loses valuable information. In such situations, there may be grounds for creating a hybrid deprivation matrix in which entries are normalised gaps for the cardinal dimensions and 0-1 deprivations for the rest. The monotonic M_α measures can then be computed from this matrix to obtain measures that reflect the depth of deprivation in each cardinal dimension, but follow the ordinal measurement restrictions for the remaining dimensions. In practice, though, this process may also increase the effective weight on ordinal dimensions – especially as α rises – since all deprived persons will appear to have the most severe degree of deprivation possible. As a correction, differential weights across dimensions may need to be contemplated, a possibility that will now be discussed in full generality.

* See Allison and Foster (2004) for an extended discussion of the measurement properties of self reported health.

8. GENERAL WEIGHTS

By using a poverty measurement methodology based on deprivation counts and simple averages, we have thus far implicitly assigned an equal weight $w_j = 1$ to each dimension j. This is appropriate when there are no compelling reasons to consider one dimension to be more important than another, or when the dimensions have been intentionally chosen such that they are of relatively equal importance. As Atkinson *et al* observe, "the interpretation of the set of indicators is greatly eased where the individual components have degrees of importance that, while not necessarily exactly equal, are not grossly different" (2002, p. 25; see also Atkinson 2003 p. 58).

Yet sometimes there are reasonably convincing arguments for according dimensions variable weights. It could be argued that the choice of relative weights of dimensions is a normative value judgement, and should be open to public debate and scrutiny: "It is not so much a question of holding a referendum on the values to be used, but the need to make sure that the weights – or ranges of weights – used remain open to criticism and chastisement, and nevertheless enjoy reasonable public acceptance" (Foster and Sen (1997)). In what follows, we will not discuss how the weights might be chosen, but only how they might be applied within the identification strategy and aggregate measures developed in this paper. Clearly, in practical applications, it is also desirable to run robustness tests on any weights that are used (Foster McGillivray and Seth, 2007).

Let w be a d dimensional row vector of positive numbers summing to d, whose j^{th} coordinate w_j is the weight associated with dimension j. Define $g^\alpha = [\, g_{ij}^\alpha]$ to be the $n \times d$ matrix whose typical element is $g_{ij}^\alpha = w_j((z_j\text{-}y_{ij})/z_j)^\alpha$ whenever $y_{ij} < z_j$, while $g_{ij}^\alpha = 0$ otherwise. From the rows g_i^0 of the weighted deprivation matrix g^0, construct the vector c of weighted deprivation counts, whose i^{th} entry $c_i = |g_i^0|$ is the sum of weights for the dimensions in which i is deprived. Each c_i varies between 0 and d, and so the associated dimensional cutoff is taken to be a real number k satisfying $0 < k \leq d$. The generalised dual cutoff identification method ρ_k is defined by $\rho_k\,(y_i; z) = 1$ whenever $c_i \geq k$, and $\rho_k(y_i; z) = 0$ whenever $c_i < k$; in other words, if the deprivation indicator c_i satisfies $c_i \geq k$, then person i is identified as being poor; otherwise, i is not poor. As before, the censored versions $c(k)$ and $g^\alpha(k)$ replace the data of the non-poor persons with 0. If $k = \min\{w_j\}$, we obtain the union identification case, while if $k = d$, the intersection; thus the ρ_k method of identification includes both of these methods. Notice that the specification $w_j = 1$ for $j = 1,\ldots, d$ corresponds to the previous case where each dimension has equal weight and the dimensional cutoff k is an integer. The specification $w_1 = d/2$ and $w_2 = \ldots = w_d = d/\{2(d\text{-}1)\}$ is an example of a *nested* weighting structure, in which the overall weight is first split equally between dimension 1 and the remaining (d-1) dimensions, and then the weight accorded the second group is allocated equally across the (d-1) dimensions. A cutoff of $k = d/2$, for example, would then identify as poor anyone who is either deprived in dimension 1 or in all the remaining dimensions.

We can revise the definition of each of our multidimensional poverty indices to accommodate general weights. The headcount ratio is $H = q/n$, where q is the number of poor persons identified by ρ_k. For the adjusted headcount, we define the average deprivation share by $A = |c(k)|/(qd)$, so that $M_0 = HA = \mu(g^0(k))$, analogous to the equally weighted definition above. The adjusted poverty gap can be expressed in terms of the average gap $G = |g^1(k)|/|g^0(k)|$ or directly in terms of the matrix $g^1(k)$ as follows: $M_1 = HAG = \mu(g^1(k))$. Analogous definitions for the adjusted M_2 measure are $S = |g^2(k)|/|g^0(k)|$ and hence $M_2 = HAS = \mu(g^2(k))$. In general the definition for the family of adjusted *FGT* measures is given by $M_\alpha = \mu(g^\alpha(k))$ for $\alpha \geq 0$. It is an easy matter to verify that each of these indices satisfies the same properties in the present context as they did with equal weights.*

9. ILLUSTRATIVE EXAMPLES

We now illustrate the measurement methodology and its variations, using data from Indonesia and the United States.

9.1 *United States*

To estimate multidimensional poverty in the US we use data from the 2004 National Health Interview Survey† on adults aged 19 and above (n = 45,884). We draw on four variables: (1) income measured in poverty line increments and grouped into 15 categories, (2) self-reported health, (3) health insurance, and (4) years of schooling. For this illustration, we assume that all variables are ordinal and therefore restrict

* Note that, in principle, a different set of weights could be used for identification and aggregation. It is also clear that when general weights are used, individual poverty measures may no longer satisfy anonymity.

† US National Center for Health Statistics (2004b)

consideration to H and M_0. The dimensional cutoffs are as follows: if a person (1) lives in a household falling below the standard income poverty line, (2) reports 'fair' or 'poor' health, (3) lacks health insurance, or (4) lacks a high school diploma, then the person is considered to be deprived in the respective dimension.* The population is partitioned into four groups: Hispanic/Latino, (Non-Hispanic) White, (Non-Hispanic) Black/African American and Other. Table 1 shows the percentages of the population deprived in each of the dimensions while Table 2 presents the distribution of deprivation counts.

Table 1: Incidence of Deprivations in US

Deprivation	Percentage of Population	Number of People
Income	12.1%	5552
Health	12.8%	5855
H. Insurance	18.3%	8405
Schooling	18.6%	8510

Table 2: Distribution of Deprivation Counts in US

Number of Deprivations	Percentage of Population	Number of People
1	23.82%	10928
2	11.67%	5353
3	4.27%	1960
4	0.44%	203

Table 3 presents the traditional income poverty headcount (the share of the population below the income cutoff), and the multidimensional measures H and M_0, where the latter are evaluated using $k = 2$ and equal weights. Column 3 gives the population share in each group while Column 5 presents the share of all income poor people found in each group. Comparing these two columns, we see that the incidence of income poverty is disproportionately high for the Hispanic and African-American populations. Moving now to the multidimensional headcount ratio H, column 7 gives the percentage of all multidimensionally poor people who fall within each group. The percentage of the multidimensionally poor who are Hispanic is much higher than the respective figure in column 5, while the percentage who are African-American is significantly lower, illustrating how our multidimensional approach to identifying the poor can alter the traditional, income-based poverty profile. Whereas column 7 gives

* Precise definitions of the indicators and their respective cutoffs appear in the Appendix.

the distribution of poor people across the groups, column 9 lists the distribution of *deprivations* experienced by the poor people in each group. The resulting figures for M_0 further reveal the disproportionate Hispanic contribution to poverty that is evident in this dataset.

Table 3: Profile of US Poverty by Ethnic/Racial Group

1 Group	2 Population	3 Percentage Contrib.	4 Income Poverty Headcount	5 Percentage Contrib.	6 H	7 Percentage Contrib.	8 M_0	9 Percentage Contrib.
Hispanic	9100	19.8%	0.23	37.5%	0.39	46.6%	0.23	47.8%
White	29184	63.6%	0.07	39.1%	0.09	34.4%	0.05	33.3%
Black	5742	12.5%	0.19	20.0%	0.21	16.0%	0.12	16.1%
Others	1858	4.1%	0.10	3.5%	0.12	3.0%	0.07	2.8%
Total	**45884**	**100.0%**	**0.12**	**100.0%**	**0.16**	**100.0%**	**0.09**	**100.0%**

Why does multidimensional poverty paint such a different picture? In Table 4, we use our methodology to identify the dimension-specific changes driving the variations in M_0. The final column of Table 4 reproduces the group poverty levels found in Column 8 of Table 3, while the rows break these poverty levels down by dimension. We use formula (1b), which in the present case becomes $M_0 = \Sigma_j H_j /d$, where H_j is the share of the respective population that is both poor and deprived in dimension j. The first row gives the decomposition for the Hispanic population, with column 2 reporting that 20% of Hispanics are both multidimensionally poor and deprived in income. Column 6 has the overall M_0 for Hispanics, which is simply the average of H_1 through H_4. The second row expresses the same data in percentage terms, with column 2 providing the percent contribution of the income dimension to the Hispanic level of M_0 or, alternatively, the percentage of all deprivations experienced by the Hispanic poor population that are income deprivations. Notice that for Hispanics, the contribution from health insurance and schooling is quite high, whereas the contribution of income is relatively low. In contrast, the contribution of income for African-Americans is relatively high. This explains why, in comparison to traditional income based poverty, the percentage of overall multidimensional poverty originating in the Hispanic population rises, while the contribution for African-Americans is lower. The example shows how the measure M_0 can be readily broken down by population subgroup and dimension to help explain its aggregate level.

Table 4: Contribution of Dimensions to Group M_0

1	2	3	4	5	6
Group	***H_1 Income***	***H_2 Health***	***H_3 H. Insurance***	***H_4 Schooling***	***M_0***
Hispanic	0.200	0.116	0.274	0.324	0.229
Percentage Contrib.	21.8%	12.7%	30.0%	35.5%	100%
White	0.045	0.053	0.043	0.057	0.050
Percentage Contrib.	22.9%	26.9%	21.5%	28.7%	100%
African-American	0.142	0.112	0.095	0.138	0.122
Percentage Contrib.	29.1%	23.0%	19.5%	28.4%	100%
Others	0.065	0.053	0.071	0.078	0.067
Percentage Contrib.	24.2%	20.0%	26.5%	29.3%	100%
Overall	0.089	0.073	0.096	0.121	0.095
Percentage Contribution	23.4%	19.3%	25.4%	31.9%	100%

9.2 *Indonesia*

The data for this example are drawn from the Rand Corporation's 2000 Indonesian Family Life Survey (Strauss, *et. al.*, 2004). Our sample consists of all adults aged 19 years and above (n = 19,752). We use d = 5 dimensions: (1) expenditure measured in Rupiah, (2) health measured as body mass index or *BMI*, in kg/m^2, (3) years of schooling, (4) drinking water, and (5) sanitation. For purposes of illustration, we make assumptions regarding the measurement properties of the dimensional variables, namely, that the first three are cardinal and the remaining two are ordinal.[*] The dimensional cutoffs are as follows: if a person (1) lives in a household with expenditures below 150,000 Rupiah, (2) has a BMI of less than 18.5 kg/m^2, (3) has fewer than five years of schooling[†] (4) lacks access to piped water or protected wells, or (5) lacks access to private latrines, then the person is deprived in the respective dimension.[‡]

[*] Strictly speaking the remaining variables are categorical variables with 10 to 11 categories each, and for illustrative purposes a plausible ordering has been selected for every dimension (Alkire and Foster 2007), Note, that as long as the categories below and above the poverty cutoff are unchanged, any alternative orderings would yield the same results for any measure that 'dichotomises' these variables.

[†] For simplification, we are ignoring the fact that at higher levels, body mass index is not positively associated with health. In the sample 610 individuals had BMI > 30 (obese); of these 214 did not experience any of the 7 remaining deprivations, 133 experienced one deprivation, and 162 obese persons (0.8% of the population) experienced 3 or more deprivations.

[‡] Precise definitions and justifications of variables and cutoffs are presented in the Appendix.

Table 5: Incidence of Deprivations

Deprivation	Percentage of Population	Number of People
Expenditure	30.1%	5952
Health (*BMI*)	17.5%	3458
Schooling	36.4%	7188
Drinking Water	43.9%	8676
Sanitation	33.8%	6681

Table 6: Distribution of Deprivation Counts

Number of Deprivations	Percentage of Population	Number of People
One	26%	5079
Two	23%	4488
Three	17%	3306
Four	8%	1588
Five	2%	326

Summary statistics presented in Table 5 show that the percentage of people deprived in each dimension ranges from 17% to 44%. Table 6 shows the percentage of the population who experience only one deprivation (26%), exactly two deprivations (23%), and so on up to five deprivations (2%) giving quite a bit of variation.

Moving to identification, Table 7 provides the number and percentage of people who would be identified as poor for each value $k = 1,\ldots,5$. When $k = 1$ (union identification), 74.7% of the population is identified as poor. When $k = 5$ (intersection identification), only 1.6% of the population is considered to be poor. Intervening values of k enable us to identify people as poor who are deprived in some but not all dimensions. The number of people identified as poor declines as the required number of deprivations increases, but at a decreasing rate.

Table 7: Identification as Cutoff *k* Is Varied

Cutoff *k*	Percentage of Population	Number of People
1	74.9%	14787
2	49.2%	9708
3	26.4%	5220
4	9.7%	1914
5	1.7%	326

In order to illustrate M_α for $\alpha = 0$,1, and 2, we first reduce consideration to the three cardinal dimensions, namely, expenditure, health, and schooling. Table 8 presents poverty levels for all values of k. Recall that each of the M_α is derived from a combination of the partial indices H, A, G, and S.

Table 8: Multidimensional Poverty Measures: Cardinal Variables and Equal Weights

Measure	$k = 1$ (Union)	$\boldsymbol{k = 2}$	$k = 3$ (Intersection)
H	0.577	**0.225**	0.039
M_0	0.280	**0.163**	0.039
M_1	0.123	**0.071**	0.016
M_2	0.088	**0.051**	0.011

We see from Table 8 that when $k = 2$, the value of the headcount ratio is 0.225, and the value of $M_0 = HA$ is 0.163; M_0 departs from H according to the level of A. In the present case, $A = M_0 /H = 0.72$, which indicates that 83% of the poor are deprived in exactly two dimensions, while the remaining 17% are deprived in all three. Note that M_0 and H coincide when all poor persons are deprived in d dimensions, as always occurs with the intersection method. Moving from M_0 to $M_1 = HAG$, the relevant factor is the average gap, which is $G = M_1 / M_0 = 0.44$ in the present case. This indicates that the average achievement of a poor person in a deprived state is 56% of the respective cutoff; if all deprived achievements were 0 and hence G were 1, then M_1 and M_0 would have the same value. $M_2 = HAS$ shows a further decrease from M_1 (0.051 rather than 0.071), and reflects the severity of poverty S. If all normalized gaps greater than zero were identical, we would expect S to equal G^2 (or 0.19 in this case). Instead, $S = 0.31$, and this larger value reflects the inequality among deprived states of the poor.

Table 9 presents a regional profile of poverty in Indonesia made possible by the fact that each of the poverty measures satisfies decomposability. We evaluate poverty for five regions using cardinal variables and choosing the intermediate level $k = 2$ as the cutoff. To begin with, comparison of columns 3 and 5 reveals that there is a disproportionately higher incidence of poverty in Bali and Sulawesi, and a correspondingly lower incidence in Sumatra. The average deprivation share A (column 12) varies only slightly across regions in this specific example and consequently the regional percentage contributions for H are nearly identical to the respective contributions for M_0 (columns 5 and 7). M_1 reflects the increased depth of deprivations in Bali and Sulawesi as compared with Sumatra and Kalimantan, due to variations in G (column 13). In Bali and Sulawesi, G is 0.49; for Sumatra and Kalimantan, 0.40 and 0.41 respectively. Similarly M_2 reflects the increased severity S of deprivations in Bali and Sulawesi, due to the unequal distribution of deprivations (beyond the depth of deprivations, that was already captured in M_1), while Sumatra and Kalimantan have correspondingly decreased levels of S and M_2.

Table 9: Profile of Poverty by Region: Cardinal Variables, Equal Weights, and k = 2

1	2	3	4	5	6	7	8	9	10	11	12	13	14
Region	**Popu-lation**	**% Contri-Bution**	***H***	**% Contri-bution**	**M_0 (HA)**	**% Contri-bution**	**M_1 (HAG)**	**% Contri-bution**	**M_2 (HAS)**	**% Contri-bution**	***A***	***G***	***S***
Sumatra	3798	19.2	0.19	16.5	0.14	16.4	0.056	15.1	0.037	14.0	0.74	0.40	0.26
Java	11928	60.4	0.22	58.9	0.16	58.9	0.068	58.2	0.049	58.3	0.73	0.43	0.31
Bali	2087	10.6	0.29	13.6	0.21	13.6	0.102	15.2	0.078	16.1	0.72	0.49	0.37
Kalimantan	827	4.2	0.22	4.2	0.16	4.1	0.065	3.8	0.045	3.6	0.73	0.41	0.28
Sulawesi	1112	5.6	0.28	7.0	0.20	7.0	0.097	7.7	0.073	8.0	0.71	0.49	0.37
All	**19752**	**100**	**0.22**	**100.00**	**0.16**	**100.00**	**0.071**	**100.00**	**0.051**	**100.00**	**0.73**	**0.44**	**0.32**

Let us now include the remaining variables. This creates a 'mixed' case with three cardinal and two ordinal variables as presented in Table 10. The first two columns report the values of H and M_0 for cutoff k = 3 with adjacent cutoffs presented for purposes of comparison. The last two columns present the values of M_1 and M_2 calculated using the procedure given above in section 7.2, with normalized gaps for the cardinal data and 0-1 'dichotomized' values otherwise. Note that it would make no difference to M_0 (or H) whether the variables were interpreted as ordinal, cardinal, or a mix of the two: the poverty levels would be unchanged, emphasizing the special versatility of M_0 in different measurement contexts.

Table 10: Multidimensional Poverty Measures:
Mixed Variables, Equal and General Weights

Cutoff	*H*	M_0	M_1	M_2
Equal Weights				
k = 2	0.49	0.27	0.19	0.18
k = 3	**0.26**	**0.18**	**0.13**	**0.12**
k = 4	0.10	0.08	0.06	0.05
General Weights				
k = 2	0.31	0.20	0.12	0.10
k = 3	**0.18**	**0.13**	**0.08**	**0.06**
k = 4	0.03	0.03	0.02	0.01

The upper half of Table 10 uses equal weights across dimensions, and the lower half employs general weights. We apply a 'nested' weighting structure which partitions the five variables into four equally weighted groups of variables, namely, expenditure, health, schooling and 'infrastructure', where weights within the final category are equally divided between the dichotomised drinking water and sanitation variables. This results in weights of w_j = 1.25 for the first three variables and w_j = 0.625 for the last two, where $|w| = d = 5$. The new weighting structure clearly affects identification and the meaning of k, but for simplicity we present the same three values of k. The new structure shifts weight from ordinal to cardinal variables, resulting in slightly more differentiation between the M_1 and M_2 values. The example illustrates how the measure M_α can be used with cardinal variables, with mixed cardinal and ordinal variables, and with general weights.

10. CONCLUDING REMARKS

This paper has proposed a new methodology for multidimensional poverty measurement consisting of: *(i)* an identification method ρ_k that extends the traditional intersection and union approaches, and *(ii)* a class of poverty measures M_α that satisfies a range of desirable properties including decomposability. Our identification step makes use of two forms of cutoffs: first, a cutoff within each dimension to determine whether a person is deprived in that dimension; second, a cutoff across dimensions that identifies the poor using a (weighted) count of the dimensions in which a person is deprived. The aggregation step employs the *FGT* measures, appropriately adjusted to account for multidimensionality. The identification method is particularly well suited for use with ordinal data, as is the first of our measures, the adjusted headcount ratio M_0. We have also provided empirical examples to show how our methodology might be used in practice.

While we have emphasized the advantages of our approach, there are several other aspects that deserve further study. First, since the identification method is based on cutoffs, it is sensitive to certain changes, but insensitive to others. For example, small changes in individual achievements around a cutoff can lead to a change in the poverty status of an individual, and can cause the poverty level to vary discontinuously in achievements.[*] It would be interesting to see whether a fuzzy approach to identification

[*] For example, using the intersection method of identification, if an achievement level of a poor person rises above the cutoff in that dimension, then the person will no longer be poor. This in turn will lead to a

might remove the discontinuity, or whether there are other modifications that might address this directly. Moreover, the poverty status of a person will be unaffected by certain large changes in achievements. Indeed, a poor person can never rise out of poverty by increasing the level of a non-deprived achievement, while a non-poor person will never become poor as a result of decrease in the level of a deprived achievement. This is perhaps not unexpected, given our interest in applying the method to ordinal data and in avoiding aggregation before identification. However, there are tensions here that should be evaluated as part of a more systematic investigation of identification methods. It would also be interesting to characterize the identification methods ρ that can be used with ordinal data, and to explore identification methods that regard group as well as individual characteristics.

Second, unlike other recent contributions, our presentation has not emphasized the potential interrelationships among dimensions that can exist when variables are cardinal. To be sure, the identification method ρ_k takes into account a rather crude form of linkage across dimensions, since a person must be deprived in k dimensions in order to be considered as poor. However, for $\alpha > 0$, the aggregation method M_α is 'neutral' in that individual i's poverty level $M_\alpha(y_i; z)$ has a vanishing cross partial derivative for any pair of dimensions in which i is deprived. It is sometimes argued that this cross partial should be positive, reflecting a form of complementarity across dimensions; alternatively, it might be negative so as to yield a form of substitutability. While M_α itself is neutral, it is a trivial matter to convert M_α into a measure that satisfies one or the other requirement: replace the individual poverty function $M_\alpha(y_i; z)$ with $[M_\alpha(y_i; z)]^\gamma$ for some $\gamma > 0$ and average across persons.[*] The resulting poverty index regards all pairs of dimensions as substitutes when $\gamma < 1$, and as complements when $\gamma > 1$, with $\gamma = 1$ being our basic neutral case. Should interrelationships among dimensions be represented in this way? When there are more than two dimensions, it might be natural to expect some pairs of dimensions to be complements and others to be substitutes, and with varying degrees and strengths. However, the γ transformation requires dimensions to be *all* substitutes or *all* complements, and with a strength that is uniform across all pairs and for all people. This seems unduly restrictive.

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discontinuous drop in virtually all multidimensional poverty indices. In the present case, as an individual rises out of poverty, the headcount will fall by $1/n$. The change in M_α is no larger than the change in H, and is weakly decreasing in α.

[*] Bourguignon and Chakarvarty (2003) present poverty indices of this kind.

Additional problems need to be faced when considering interrelationships among dimensions. There are multiple definitions of substitutes and complements, and the leading candidate – the Auspitz-Lieben-Edgeworth-Pareto (ALEP) definition – has certain difficulties (Kannai 1980). Moreover, there do not seem to be any convincing empirical procedures for determining the extent of substitutability and complementarity across dimensions of poverty. Nor has it even been established that the potential interrelationships must be reflected in an overarching methodology for evaluating multidimensional poverty. Instead, the interconnections might be the subject of separate empirical investigations that supplement, but are not necessarily part of, poverty measurement. Our methodology provides a neutral foundation upon which more refined accounts of the interconnection between dimensions can be built.

This paper leaves a number of questions for subsequent studies to address. For example it would be interesting to see whether the adjusted *FGT* measures M_α and the dual cutoff identification method ρ_k can be fully characterized. It would likewise be natural to investigate dominance conditions that would allow poverty comparisons to be robust to the choice of cutoffs or weights. Another unresolved question is whether a measure can be crafted for ordinal data that reflects the depth of dimensional deprivations. We hope that the methodology developed in this paper will be a useful touchstone for future research efforts.

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APPENDIX: DATA AND POVERTY CUTOFFS

This section presents the precise definition of the variables, and the poverty cutoffs and their justification for each dimension. The dimensions in both countries were chosen because the data are arguably related to current research on poverty, and have the requisite technical characteristics. To obtain policy relevant conclusions, the selection of dimensions would need to satisfy additional normative criteria (Robeyns, 2005; Alkire, 2008).

United States

US data were used from the 2004 National Health Interview Survey on adults aged 19 and above (n = 45,884).

1. Income

The definition of estimated earnings is described in Center for Disease Control (2005, p. 36f). The poverty cutoffs vary by household composition; the formula used are presented on http://www.census.gov/hhes/www/poverty/threshld/thresh04.html (accessed 30 Dec 2007). The data are the ratio of family income to the poverty threshold, and comprise 15 categories, 3 below the poverty line and 12 above.

2. Health:

Definition: The question had five categories and read: “Would you say your health in general is (5) excellent, (4) very good, (3) good, (2) fair, or (1) poor?” We considered those who responded ‘fair’ or ‘poor’ to be deprived in terms of self-reported health, and others to be non-poor.

3. Health Insurance:

Definition: the question read: ‘What kind of health care or health insurance does this person have?’ We considered those who responded, ‘No coverage of any type’ to be deprived in terms of health insurance, and others to be non-poor. Note that the data had been corrected, and regarded as uninsured, ‘persons who did not report having health insurance at the time of the interview under private health insurance, Medicare, Medicaid, State Children's Health Insurance Program (SCHIP), a State-sponsored health plan, other government programs, or military health plan (includes TRICARE, VA, and CHAMP-VA).' This definition of uninsured matches that used in US National Center for Health Statistics (2004a).

4. Schooling:

Definition: Education is measured by years of schooling completed. Following convention, the poverty cutoff classified those who had not completed a GED or received a high school diploma to be deprived and others to be non-poor. When conducting Spearman's rank correlations, we used the number of years of schooling through 12, but ranking a person with 12 years of schooling and no diploma as 11. Other data were coded as followed: GED=12, College no degree=13, Associate degree (occupational, technical or academic) = 14 Bachelor's degree=15, Master's degree = 16, Professional Degree (MD) or PhD = 17.

Table A1: Spearman's Rank Correlation (non-censored data), USA

	Income	Health	Schooling	Insurance
Income	1.00			
Health	0.26	1.00		
Health Insurance	0.31	0.03	1.00	
Schooling	0.47	0.28	0.24	1.00

Indonesia

The data derive from individual and household level questionnaires for adults 19 years and above.[*] Years of Schooling and Body Mass Index pertain to the individual and per capita household expenditure is calculated. The individual is ascribed the values satisfied by their household for drinking water and sanitation.

In variables 4 and 5, a plausible ordering chosen is for illustrative purposes. There are serious issues with categorical variables of how to order correctly.[†] Alternative

[*] All data and documentation can be downloaded from http://www.rand.org/labor/FLS/IFLS/ifls3.html.

[†] The category 'other' is particularly difficult; we have arbitrarily ascribed it the lowest value in all variables.

orderings could have been used, but so long as the sets below and above the poverty line remain the same, an alternative ordering will yield the same results for H and M_0 as these are based on dichotomised data.

1. Expenditure

The variable is monthly real per capita household expenditure as defined in (Strauss, *et al*., 2004) and presents. The poverty cutoff is z_1 = 150,000 (in current 2000 Rupiah) per capita. This poverty line was adopted for the same dataset by Massoumi and Lugo, and is roughly equivalent to $0.5 per day per person. Note that the Indonesian government uses different poverty lines for rural and urban areas.

2. Health: Low Body Mass Index (BMI)

Definition: Body Mass Index (BMI) is the weight in kilograms divided by the square of the height in metres (kg/m^2). The poverty cutoff z_2 = 18.5 kg/m^2. This is the standard international classification for underweight adults taken from the World Health Organisation Guidelines found on http://www.who.int/bmi/index.jsp?introPage=intro_3.html

3. Schooling

Definition: Education is measured by years of schooling completed. The poverty cutoff z_3 = 6 years of schooling, as in India primary school is completed in 6 years. This is an imperfect indicator for primary education as it does not consider those who have repeated a year of schooling.

4. Drinking water

Definition: This is based on responses to the question, "What is the main water source for drinking for this household?" The poverty cutoff used is taken from the cutoff for MDG indicator 30 (United Nations Development Group, 2003) p 64-6.[*] Note that the data do not specify whether wells and springs are protected, and so certain assumptions had to be made. There are 10 categories. We set z_4 = 9, thus have regarded as non-poor all persons obtaining piped or pumped well water, and the remainder as deprived.

[*] This approach does not regard bottled water such as Aqua/Air Mineral as clean drinking water hence we follow the convention, but acknowledge that adjustments may be required in some situations.

10. Pipe Water
9. Well/Pump (electric, hand)
8. Aqua/Air Mineral, etc
7. Well Water
6. Spring Water
5. Rain Water
4. River/Creek Water
3. Pond/Fishpond
2. Water Collection Basin
1. Other

5. Sanitation

Definition: This is based on responses to the question, "Where do the majority of householders go to the toilet?" The poverty cutoff used is taken from the cutoff for MDG indicator 31 (United Nations Development Group, 2003) p 66-8. There are 11 categories. We set $z_5 = 10$ so regard as non-poor those persons who use their own toilet (with or without a septic tank) and regard all others as deprived.

11. Own toilet with septic tank
10. Own toilet without septic tank
9. Shared toilet
8. Public toilet
7. Creek/river/ditch (without toilet)
6. Yard/field (without toilet)
5. Sewer
4. Pond/fishpond
3. Animal stable
2. Sea/lake
1. Other

Table A2: Indonesian Spearman's Rank correlation (non-censored data), Indonesia

	Expenditure	**Health (BMI)**	**Schooling**	**Drink. Water**	**Sanita-tion**
Expenditure	1.00				
Health (BMI)	0.19	1.00			
Schooling	0.35	0.14	1.00		
Drinking Water	0.26	0.15	0.26	1.00	
Sanitation	0.29	0.14	0.31	0.27	1.00